**Chapter 4: Algebra of Vectors Test A** Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**in two dimensions**

*Simple familiar*

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|  | If  = 4 − 3 and  = 2 + 4, then determine the vector 4 − 2.5 . | [1 mark]  [1 mark] | 2 |
|  | Which of the following vectors has magnitude 25?  A  B  C  D | [1 mark]  The answer is C | 1 |
|  | If  = 3 + 4and *v* *=* −4 + 5, then calculate the value of . |  | 1 |
|  | Evaluate the size of angle between   = 3 + 4 and  *=* −4 + 5,  correct to 2 decimal places. | [1 mark]  [1 mark] | 2 |
|  | Determine a unit vector perpendicular to  −3 + 4. | When two vectors are perpendicular, their dot product is zero. [1 mark]  A vector perpendicular to  is .  A unit vector perpendicular to  is . [1 mark] | 2 |
|  | TY08-03  The 3-kg object is in equilibrium on a smooth plane.  Express the magnitude of forces and in terms of *g*. | [1 mark]  [1 mark] | 2 |
|  | A cart of mass 16 kg is dragged across a horizontal floor at a constant speed of 1 m/s by a force of 100 N acting at an angle of 20° to the horizontal. Calculate the value of the coefficient of friction, leaving your answer in exact form. | [2 marks] | 2 |
|  | Two forces,  and , act simultaneously on an object.   1. Calculate the magnitude of the resultant force. 2. Determine the direction of the resultant force | 1. The resultant force is   [1 mark]  b.  The direction is 36.9° anticlockwise from [1 mark] | 2 |
|  | A windsurfer heads due east at 24 km/h with respect to the water but is pushed off course by a 15-km/h wind from the north. Determine the velocity of the windsurfer as seen by an observer on the shore. | [1 mark]  An observer would see the windsurfer travelling at 28 km/h at a bearing of   = 122°. [1 mark] | 2 |
|  | Given  *=*  + 2 and  *=* 2 + 3, determine the vector resolute of  parallel to  . | [1 mark]  [1 mark] | 2 |
|  | In the diagram shown, D is the midpoint of CB and E is the midpoint of AD. The position vectors of points A, B, C, D and E are given by , , ,  and  respectively.  TY08-1-1  Express the value of , the position vector of D, in terms of the vectors  and . | [2 marks] | 2 |
|  | Let (*t*) be a position vector of an object whose position varies with time. If   *=* 3 sin *t* *+* 3 cos *t*  then the path this object takes is:   1. a straight line 2. a parabola 3. an ellipse 4. a circle | [1 mark]  The path the object takes is a circle.  D [1 mark] | 2 |

*Complex familiar*

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|  | If  and , determine  if:   1. and  are parallel 2. and  are perpendicular 3. and  are equal in length 4. the scalar resolute of  on  is . | 1. [1 mark] 2. [1 mark]     [1 mark]  [1 mark] | 4 |
|  | Let  ***=*** 5 + 3 and  **=**  **+** 2.   1. Calculate the angle between the two vectors, in radians, to 4 decimal places. 2. Find , the angle which  makes with the *x*-axis, in radians rounded to 4 decimal places. 3. Find the vector resolute of  in the direction of . | [1 mark]      [1 mark]        [1 mark] | 3 |
|  | A 4-kg mass is placed on a smooth inclined plane and a force of magnitude 24 N acting up the plane is applied. If the plane is inclined at 30° to the horizontal, calculate:  (a) the component of the weight acting down the plane  (b) the normal reaction  (c) the resultant force. | TA8  (a) Component of weight down the plane = 4*g* sin 30° [1 mark] = 19.6 N [1 mark]  (b) *N* = 4*g* cos 30° [1 mark] *N* = 33.9 N [1 mark]  (c) *R* = 24 – 19.6 [1 mark] *R* = 4.4 N up the plane [1 mark] | 6 |
|  | Let  *=* 2 *+* 4. Determine a vector, parallel to , such that their dot product is 40. | *m* ***=*** *m*(2 *+* 4)  *m* = *m*(2 *+* 4)(2 *+* 4) = 40  [1 mark]  4*m* + 16*m* = 40  20*m* = 40  *m* = 2 [1 mark]  The new vector is 4 *+* 8. [1 mark] | 3 |

*Complex unfamiliar*

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|  | The three forces shown in the figure below are acting such that the particle is in equilibrium. Calculate the angle between the forces with magnitude *F* N and 16 N.  SM Test Yself fig 45 | If the forces are in equilibrium, their sum is 0.  [1 mark]  TA8  Using the cosine rule:  *F*2 = 162 + 202 − 2 × 16 × 20 cos 30° [1 mark]  = 101.7  *F* = 10.1 N [1 mark]  Using the sine rule:  [1 mark]  [1 mark]  *φ* = 180° − 82° = 98° [1 mark]  Thus, the angle between the force of 16 N and the force of *F* N is 98°. | 6 |
|  | Calculate the equation of the path of the *time-varying* position vector  ***=***  ***+*** 2 (*t*2 − 1). State the type of path (linear, parabolic and so on). Hence, sketch its graph, and indicate the direction of the path as *t* increases. | *x* *=*  so *t* *=*  *y* = 2(*t*2 – 1) [1 mark]  *y* = 2  =  − 2 [1 mark]  The path is hyperbolic. [1 mark]  SM Test A fig 9-06a As  increases,  decreases and  increases, hence direction as shown.  *y* = –2  [1 mark for curve, 1 mark for direction] | 5 |
|  | The top of a 10-m diving board lies over the swimming pool as illustrated below.  SM Test A fig 9-01  Sally sits 30 m away in the corner of the swimming pool and takes a bearing of 40° (N40°E) of the feet of her friend who is about to do a belly flop. State the position vector from Sally’s current position to her friend’s feet. | Assume that Sally’s position is the origin.  Then from the origin, Sally’s friend’s feet will have the following coordinates:  the *x*-coordinate will be 30 sin 40°, [1 mark] the *y*-coordinate will be 30 cos 40° [1 mark] and the *z*-coordinate will be 10. [1 mark]  The position vector is:  30 sin 40° **+** 30 cos 40° **+** 10  = 19.28 **+** 22.98 **+** 10. [1 mark] | 4 |
|  | A river flows east–west at 5 m/s. A tugboat can motor at 3 m/s, and the captain tries to motor directly across the river from south to north.   1. Draw a vector diagram. 2. Calculate the resultant speed of the tugboat. 3. Calculate the bearing of the tugboat. 4. If it took 2 minutes to reach the opposite bank, how wide is the river? 5. In 2 minutes, how far downstream would the tugboat be carried? | 1. SM Test A fig 9-10 [1 mark] 2. Resultant speed   [1 mark]   1. SM Test A fig 9-11  Bearing = N  W,  where  [1 mark]  = 59.04° [1 mark] So the bearing is N59.04°W or  300.96° true. 2. 2 minutes = 120 seconds Distance = speed × time Distance = 3 × 120 [1 mark] Distance = 360 m [1 mark] 3. Distance downstream = 5 × 120 [1 mark] Distance downstream = 600 m [1 mark] | 8 |